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Effect of Lead Time on Inventory — A Working Result

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A relationship between standard deviation of demand during lead time (normally distributed) and economic order quantity on the basis of (Q, R) inventory model with variable lead time has been developed.

INTRODUCTION

IN THE analysis of effect of lead time on inventory Das[1] has suggested a model with total Inventory cost "$T(Q)$" as a function of Q and is given by:

$$T(Q) = (AK/Q) + (HQ/2) + G(1 - NQ/A)H\sigma_L, \tag{1}$$

where

D = Demand per unit time (day), (a strictly positive random variable),
μ = Mean value of D,
σ = Standard deviation of demand (D),
A = Expected total demand for the entire planning period (year),
K = Set up cost per order,
H = Holding cost per unit per year,
Y = Number (a random variable) of lead time per year in which stock out occurs,

$E(Y) \leqslant N$, i.e. expected value of Y is less than or equal to N, where N is a given constant,

M = Mean lead times (in days),
V = Variance of lead time,
L = Total Demand during lead time

It can be shown[2] that if L is a random variable with

μ_L = Mean value of $L = M\mu$
σ_L^2 = Variance of $L = M\sigma^2 + V\mu^2$,

the optimal value of Q is determined as follows:

$$0 = \frac{\partial T(Q)}{\partial Q} = -AK/Q^2 + (H/2) - (N/A)H\sigma_L G'(1 - NQ/A), \tag{2}$$

or

$$Q^2 = \frac{2A^2K}{HA - 2NH\sigma_L G'(1 - NQ/A)}. \tag{3}$$

It is apparent that for most distributions of lead time demand an explicit expression which gives the optimal value of Q, is seldom available from (3).

The present analysis seeks to suggest a graphical procedure for finding Q^*, i.e. the optimal value of Q from (3) under the assumption that the demand during lead time follows a normal distribution, and under other assumptions stated by Das.[1]

ANALYSIS

It follows from (3) that

$$Q = \sqrt{\frac{2A^2K}{HA - 2NH\sigma_L G'(1 - NQ/A)}},$$

or

$$Q = \sqrt{\frac{2AK}{H}} \sqrt{\frac{A}{A - 2N\sigma_L G'(1 - NQ/A)}}. \tag{4}$$

Let $F(X)$ be cumulative density function of standardised normal variate $X = (L - \mu_L)/\sigma_L$,

$$G(X) = F^{-1}(X)$$

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$$F[G(X)] = X. \tag{5}$$

Differentiation of (5) gives,

$$F'[G(X)] . G'(X) = 1,$$

where

$$\left\{ \begin{array}{l} G'(X) \text{ stands for } \dfrac{dG(X)}{dX} \\ \text{and} \\ F'(X) \text{ stands for } \dfrac{dF(X)}{dX} \end{array} \right\}$$

$G'(X) = \dfrac{1}{F'[G(X)]}$, provided $F'[G(X)] \neq 0$.

Since $F'[G(X)]$ is ordinate of normal curve at $G(X)$, then

$$G'(X) = \frac{1}{\text{Ordinate } G(X)},$$

hence

$$G'(1 - NQ/A) = \frac{1}{\text{Ordinate } G(1 - NQ/A)}. \tag{6}$$

Substituting (6) in (4) we get,

$$Q = \sqrt{\frac{2AK}{H}} \cdot \sqrt{\frac{A}{A - \dfrac{2N\sigma_L}{\text{Ordinate } G(1 - NQ/A)}}}$$

$$= \sqrt{\frac{2AK}{H}} \cdot \sqrt{\frac{A \text{ Ordinate } G(1 - NQ/A)}{A \text{ Ordinate } G(1 - NQ/A) - 2N\sigma_L}},$$

$$Q = \sqrt{\frac{2AK}{H}} \cdot \sqrt{\frac{\text{Ordinate } G(1 - NQ/A)}{\text{Ordinate } G(1 - NQ/A) - (2N/A)\sigma_L}}. \tag{7}$$

Procedure suggested for calculating the optimal value of Q

The equation (7) above can be solved with the help of trial and error method. For finding the solution, the following procedure may be followed:—

(a) Give a few values to Q on left hand side (LHS) and find corresponding values of right hand side (RHS) from (7) for a particular value of σ_L. (N.B.: The same arbitrary values for "Q" have to be given on LHS and RHS).

(b) Plot these points, taking LHS values on one axis and RHS on the other axis for different values of σ_L.

(c) The points, where the line $Q = \omega$ cuts these curves, will give optimal values of Q, i.e. Q^* at different values of σ_L.

(d) Plot a second graph taking σ_L as Q-axis and Q^* as ω-axis.

(e) If mean and variance of lead time are given (M_1, V_1) in one situation and (M_2, V_2) in the other, then,

$$\sigma_{L_1} = \sqrt{M_1\sigma^2 + V_1\mu^2}$$

and

$$\sigma_{L_1} = \sqrt{M_2\sigma^2 + V_2\mu^2}.$$

Steps "(a)–(c)" yield the optimal values of Q, i.e. Q_1^* and Q_2^* with respect to σ_{L_1} and σ_{L_2}.

(f) Substituting Q_1^* and Q_2^* in (1) the optimal total cost values in both the situation are given by, $T_{C_1}^*$ and $T_{C_2}^*$.

Now, if $T_{C_1}^* > T_{C_2}^*$ (the second situation which has mean and variance of lead time as M_2 and V_2 is preferred over the first), and if $T_{C_1}^* < T_{C_2}^*$ (the first situation with mean and variance of lead time as M_1 and V_1 is preferred over the second).

Procedure suggested for calculating the service level

The "service level" is the percentage of demand to be filled routinely from on-hand inventory.

$$\text{Service level } (S_L) = \frac{\text{Shipments}}{\text{Orders placed}} \tag{8}$$

$$= 1 - \frac{\text{Orders not met}}{\text{Orders placed}}$$

$$= (1 - \text{Probability of stock out}).$$

$$\text{Service level } (S_L) = (1 - NQ/A).$$

Finally,

$$S_L \text{ in percentage} = (1 - NQ/A)100. \tag{9}$$

The optimal percentage value of the service level is obtained by substituting Q^* for Q in (9).

NUMERICAL ILLUSTRATION

Let $A = 5000$, $H = 10$, $K = .5$ and $N = 1$.

Substituting above set of parameters in (7) we get,

$$Q = 70.7\sqrt{\frac{\text{Ordinate } G(1 - 0.0002Q)}{\text{Ordinate } G(1 - 0.0002Q) - 0.0004}}.$$

Let

$$\sigma_L = 50, 100, 150, 200, 250, 300, 350, 400, 500, 600.$$

(a) After applying the graphical procedure we will get 10 different curves (Figure 1), (since there are 10 values of σ_L), representing the nature of the inventory system.

(b) The line $Q = \omega$ will cut the curves in Figure 1 according to Table 1, represented by Figure 2.

(c) By substituting the different values of Q^* so obtained in (9), we will get the corresponding optimal values of service level in percentage as per Table 2, represented by Figure 3.

CONCLUSION

Thus, in a specific situation faced by the Materials Manager, a working chart can be constructed by taking σ_L as Q-axis and E.O.Q. as ω-axis. Note that changes in

TABLE 1

σ_L	50	100	150	200	250	300	350	400	500	600
Q^*	95	124	162	210	265	320	384	450	612	800

TABLE 2

Q^*	95	124	162	210	265	320	384	450	612	800
S_L in %	98.1	97.5	96.7	95.8	94.7	93.6	92.3	91.0	87.7	84.0

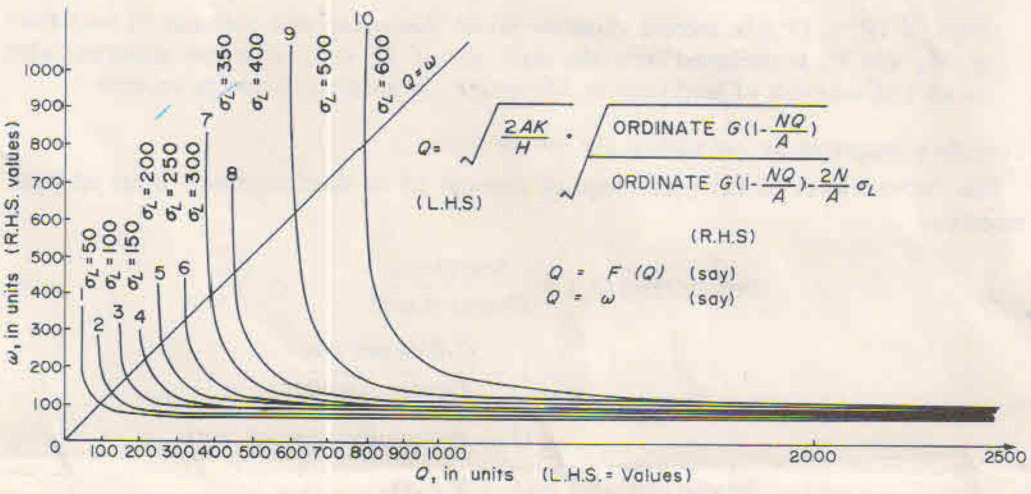

FIG. 1. *A graphical representation of equation* (7) *at different values of* σ_L.

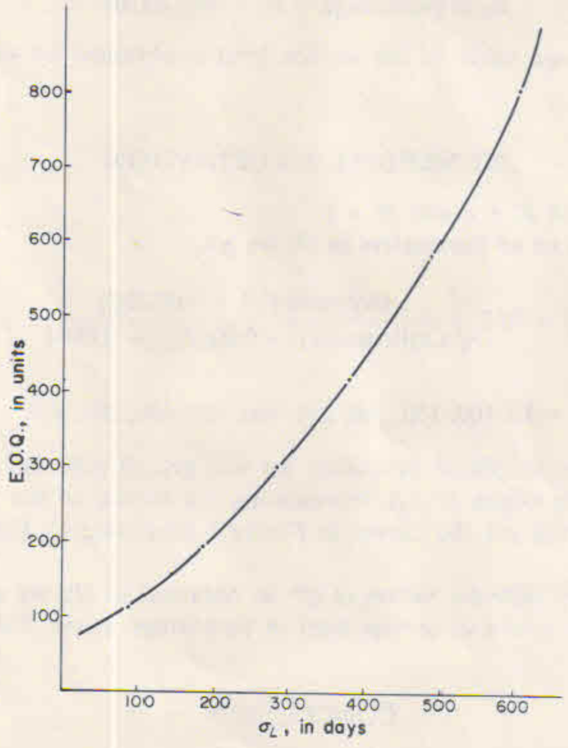

FIG. 2. *Graph showing the relation between optimal values of ordering quantity,* i.e. (Q^*) *and the standard deviation of demand during lead time* (σ_L).

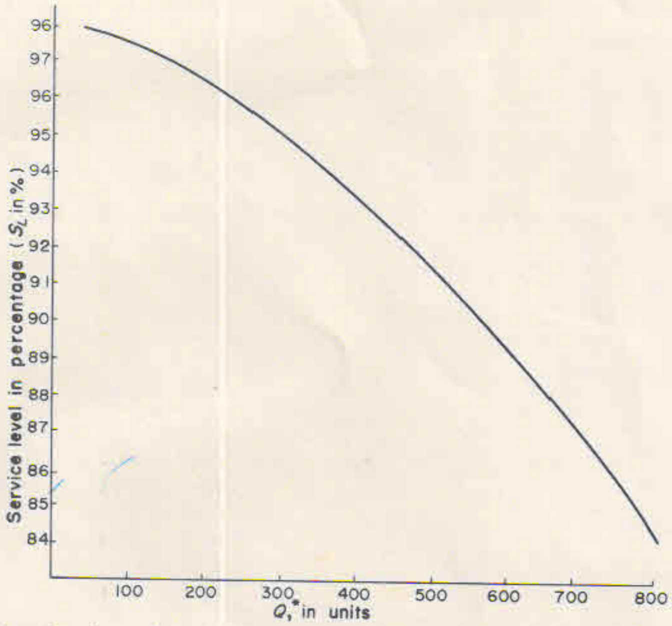

FIG. 3. *Graph showing the relation between optimal values of ordering quantity* i.e. (Q*) *and the service level* (S_L) *in percentage.*

the nature of supply and demand will bring about a change in σ_L and as such alter the values of E.O.Q. Further, since the increase in σ_L does not bring about a proportionate increase in Q^* the service level goes down with every increase in σ_L. Hence after keeping an eye on all these factors, the Materials Manager can decide his optimal course of action.

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